

The last word in strong correlations

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In the Fractional Quantum Hall Effect (FQHE), in the noninteracting limit, only a fraction ν of the Lowest Landau Level (LLL) is occupied, producing a huge degeneracy. Interactions lift this degeneracy and mix in higher LL's. In the limit in which we ignore all but the LLL (i.e., let the inverse electron mass $\frac{1}{m} \rightarrow \infty$), the kinetic energy is an irrelevant constant and the ratio of potential to kinetic energy is essentially infinite, making this the most strongly correlated problem imaginable. I give a telegraphic review of the Hamiltonian Theory of the FQHE developed with Ganpathy Murthy that deals with this problem with some success. A nodding acquaintance with FQHE physics is presumed.

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I. INTRODUCTION

The first breakthrough in the FQHE came from the approach pioneered by Laughlin¹ and extended primarily by Jain² and consists of writing down inspired trial wavefunctions.

The hamiltonian approach³ is a complimentary one that begins with the microscopic hamiltonian for interacting electrons and tries to obtain a satisfactory description of the underlying physics through a sequence of transformations and approximations. It gives a concrete operator realization of many heuristic pictures that have been espoused and makes precise under what conditions and in what sense these pictures are valid. It allows one to compute to reasonable accuracy (10 – 20%) a large number of quantities such as gaps, relaxation rates, polarizations etc., at zero and nonzero temperatures, at equal and unequal times, and even permits a crude model of disorder^{4,5}.

The hamiltonian approach differs from the Chern-Simons (CS) approach which has similar objectives. There one makes a singular gauge transformation on the electronic wavefunction (in the operator approach)⁶ or couples electrons to a Chern-Simons gauge field (in the path integral approach)^{7,8}, leading in either case to a composite particle which is the union of an electron and some number of point flux tubes.

For the Laughlin fractions

$$\nu = \frac{1}{2s+1} \quad (1)$$

where there are $2s+1$ external flux quanta per electron, one can either attach $2s+1$ flux tubes to each electron in opposition to the applied field, and turn it into a boson in zero (mean) field that becomes a superfluid⁷ or add $2s$ opposing flux quanta to turn it into a composite fermion that sees a net of one flux quantum and fills the LLL of CF's.

For the more general Jain fractions

$$\nu = \frac{p}{2ps+1}, \quad (2)$$

where there are $2s + \frac{1}{p}$ external flux quanta per electron,

the only viable option is to attach $2s$ quanta to each electron, producing CF's that see $\frac{1}{p}$ flux quanta each and fill exactly p LL's. This leads to the wavefunction

$$\Psi = \mathcal{P} \prod_{j<i} (z_i - z_j)^{2s} \times \chi_p(z, \bar{z}) \exp(-\sum_i |z_i|^2/4l^2). \quad (3)$$

Here $l = \sqrt{\hbar/eB}$ is the electron's magnetic length. The Jastrow factor up front comes from the flux attachment, $\chi_p(z, \bar{z})$ is the wavefunction for p filled CF LL's, and the exponential factor is ubiquitous. The operator \mathcal{P} projects away all the \bar{z} dependence of χ_p for $p > 1$ so that Ψ resides in the electronic LLL. In fact another projection has already been done: the flux- attaching gauge transformation produces only the *phase* of the Jastrow factor and the analytic zeros of the final Jastrow factor come only upon projecting away the $|z_i - z_j|^{-2s}$ dependence of the phase.

The $2s$ -fold vortex at the location of every electron causes a charge deficit of $-2pse/(2ps+1)$, as can be shown by Laughlin's plasma analogy or flux threading argument. Together with the charge e of the electron, this implies a screened quasiparticle of charge $e^* = e/(2ps+1)$. It has no memory of the electron mass m , is sustained by just the interactions and lives entirely in the LLL. This is the physical CF that we want to access and describe in our approach.

A parameter that plays a central role in this article is

$$c^2 = \frac{2ps}{2ps+1} \leq 1. \quad (4)$$

Henceforth we will focus on $s = 1$ so that just two flux quanta are attached to each electron in the CS approach and a double vortex appears at each electron's location in the wave function.

The composite fermions of CS theory have a non-degenerate ground state at mean-field level, which is their main allure. This state is usually gapped. The concept is however very effective even for the gapless case $\nu = \frac{1}{2}$ ($c = 1, p = \infty$), where many phenomena pertaining to an over-damped mode, coupling to surface acoustic waves, and the compressibility are successfully described⁹.

On the other hand, these CS fermions do not exhibit in any transparent way the quasiparticle properties (such as charge e^* or effective mass m^*) deduced from trial wavefunctions, do not reside in the LLL and indeed have a singular limit as the electron mass $m \rightarrow 0$.

The Hamiltonian Theory which Murthy and I developed over the years³ works in the LLL all along and addresses some of these issues. We asked how one is to incorporate into the theory what the interpretation of the excellent wavefunctions tells us, namely that "an electron is bound to two vortices." But what does that even mean? Vortices are not elementary particles with their own degrees of freedom or dynamics, they are zeros in a wavefunction for electrons! In our earlier work we dealt with this by introducing extra plasmonic degrees of freedom *à la* Bohm-Pines¹⁰ in an enlarged Hilbert space with some constraints. We wrote down a product wavefunction (for electrons and plasmons), which upon projection to the (constrained) physical sector yielded the Jain wave function including the complete analytic Jastrow factor *with its multiple zeros* and not just its phase. This approach evolved and got refined over time. Here I fast-forward to the final version which is an efficient starting point for many calculations, directing the reader eager for details to Ref.³.

II. THE HAMILTONIAN APPROACH

The primordial hamiltonian in terms of electronic variables (which carry the subscript e to distinguish them from other coordinates to be introduced shortly) is:

$$H = \sum_j \frac{\eta_{ei}^2}{2ml^4} + \frac{1}{2} \sum_{i,j,\mathbf{q}} v(q) e^{i\mathbf{q} \cdot (\mathbf{r}_{ei} - \mathbf{r}_{ej})} \equiv H_0 + V \quad (5)$$

where η_{ei} is the i -th electron's cyclotron coordinate that lets it move up and down Landau levels and \mathbf{r}_{ei} is its coordinate in the plane.

Projecting to the LLL one drops the first (kinetic energy) term and makes the replacement

$$e^{i\mathbf{q} \cdot \mathbf{r}_e} = e^{i\mathbf{q} \cdot (\boldsymbol{\eta}_e + \mathbf{R}_e)} \rightarrow \langle e^{i\mathbf{q} \cdot \boldsymbol{\eta}_e} \rangle_{\text{LLL}} e^{i\mathbf{q} \cdot \mathbf{R}_e} = e^{-q^2 l^2 / 4} \cdot e^{i\mathbf{q} \cdot \mathbf{R}_e} \quad (6)$$

where \mathbf{R}_e is the electronic guiding center coordinate. Thus the projected Hamiltonian is

$$\bar{H} = \frac{1}{2} \sum_{i,j,\mathbf{q}} v(q) e^{-q^2 l^2 / 2} e^{i\mathbf{q} \cdot (\mathbf{R}_{ei} - \mathbf{R}_{ej})} \quad (7)$$

(While we limit ourselves to the LLL in this article, one can retain the $\boldsymbol{\eta}_e$ in Eqn. (5) to study, for example, LL mixing as we did in Ref.¹¹.)

Although we do not have a kinetic energy term, the problem is hard because the components of \mathbf{R}_e do not commute

$$[R_{ex}, R_{ey}] = -il^2. \quad (8)$$

Thus R_{ex}, R_{ey} are conjugate variables. Now a full fledged fermion in $d = 2$ will have two coordinates and

two momenta, i.e., two conjugate pairs. The LLL projected electron is like half a fermion and this is what makes the analysis difficult. So we introduce another conjugate pair of "vortex" guiding center coordinates which we will define by their commutation relations:

$$[R_{vx}, R_{vy}] = \frac{il^2}{c^2} \quad \text{where } c^2 = \frac{2p}{2p+1}. \quad (9)$$

Thus the vortex describes a particle whose charge $-\frac{2p}{2p+1}$ in electronic units is exactly that of the vortices in the Jastrow factor. It too is just half a particle like the projected electron.

We want these extraneous coordinates to commute with everything electronic i.e.,

$$[\mathbf{R}_e, \mathbf{R}_v] = 0. \quad (10)$$

Now the point is that we can accommodate both \mathbf{R}_e and \mathbf{R}_v and their algebra very neatly into the Hilbert space of a regular two-dimensional fermion, which is going to be our composite fermion. This fermion is bathed in the reduced field seen by a e^* object. From its position vector \mathbf{r} and kinetic momentum $\boldsymbol{\Pi} = \mathbf{p} - e\mathbf{A}$ we can construct its guiding center and cyclotron coordinates (which carry no subscripts like e or v) that obey

$$[\eta_x, \eta_y] = il^{*2} = \frac{il^2}{1 - c^2} \quad (11)$$

$$[R_x, R_y] = -il^{*2}. \quad (12)$$

In terms of these two conjugate pairs \mathbf{R}_e and \mathbf{R}_v can be represented as follows:

$$\mathbf{R}_e = \mathbf{R} + \boldsymbol{\eta} c \quad (13)$$

$$\mathbf{R}_v = \mathbf{R} + \boldsymbol{\eta} / c. \quad (14)$$

An equivalent representation in terms of \mathbf{r} and $\boldsymbol{\Pi}$, the CF coordinate and velocity operators, is

$$\mathbf{R}_e = \mathbf{r} - \frac{l^2}{(1 + c)} \hat{\mathbf{z}} \times \boldsymbol{\Pi}, \quad (15)$$

$$\mathbf{R}_v = \mathbf{r} + \frac{l^2}{c(1 + c)} \hat{\mathbf{z}} \times \boldsymbol{\Pi}. \quad (16)$$

The inverse transformation is illuminating:

$$\mathbf{R} = \frac{\mathbf{R}_e - c^2 \mathbf{R}_v}{1 - c^2} \quad (17)$$

$$\boldsymbol{\eta} = \frac{c}{1 - c^2} (\mathbf{R}_v - \mathbf{R}_e) \quad (18)$$

The first equation could have been guessed: it says that the CF guiding center is the weighted sum of its charged parts. The second equation can be found by demanding that $\boldsymbol{\eta}$ be linear in \mathbf{R}_e and \mathbf{R}_v , commute with \mathbf{R} , and have an overall scale that produces the right commutator.

Consider Eqns. (15,16) when $\nu = \frac{1}{2}$ or $c = 1$, and $\boldsymbol{\Pi} = \mathbf{p}$ (the CF sees no field). We see that \mathbf{R}_e and \mathbf{R}_v

are located on either side of \mathbf{r} separated by $\hat{\mathbf{z}} \times \mathbf{p} l^2$. This is the operator realization of Read's dipole picture¹².

Ignoring the zero point energy, here is where we stand in the LLL sector:

$$\begin{aligned} \bar{H} &= \frac{1}{2} \sum_{i,j,\mathbf{q}} v(q) e^{-q^2 l^2/2} e^{i\mathbf{q} \cdot (\mathbf{R}_{ei} - \mathbf{R}_{ej})} \\ &= \frac{1}{2} \sum_{i,j,\mathbf{q}} v(q) e^{-q^2 l^2/2} \exp(i\mathbf{q} \cdot [(\mathbf{R}_i - \mathbf{R}_j) + c(\boldsymbol{\eta}_i - \boldsymbol{\eta}_j)]) \end{aligned} \quad (19)$$

While it is true that we have managed to get rid of the electron mass m and isolate the LLL cleanly, the reader may ask what we have gained, since *algebraically* the problem is the same as in electronic coordinates. *The answer is that now there is a natural nondegenerate HF ground state in the enlarged CF space.* This is because the HF hamiltonian is now written in terms of CF operators \mathbf{R} and $\boldsymbol{\eta}$ and the particle density is just right to fill the lowest p CF-LL's. This key step opens up all the usual approximation schemes.

From \mathbf{R}_e and \mathbf{R}_v we can form the corresponding electron and vortex densities:

$$\bar{\rho}(\mathbf{q}) = \sum_j e^{-i\mathbf{q} \cdot \mathbf{R}_e} \quad (21)$$

and

$$\bar{\chi} = \sum_j e^{-i\mathbf{q} \cdot \mathbf{R}_v}. \quad (22)$$

These obey

$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] = 2i \sin \left[\frac{(\mathbf{q} \times \mathbf{q}') l^2}{2} \right] \bar{\rho}(\mathbf{q} + \mathbf{q}') \quad (23)$$

which was thoroughly exploited in¹³ and

$$[\bar{\chi}(\mathbf{q}), \bar{\chi}(\mathbf{q}')] = -2i \sin \left[\frac{l^2(\mathbf{q} \times \mathbf{q}')}{2c^2} \right] \bar{\chi}(\mathbf{q} + \mathbf{q}'). \quad (24)$$

The electron and vortex densities commute since \mathbf{R}_e and \mathbf{R}_v do.

The mathematical problem we face is then summarized by the following:

$$\bar{H} = \frac{1}{2} \sum_{\mathbf{q}} v(q) e^{-q^2 l^2/2} \bar{\rho}(\mathbf{q}) \bar{\rho}(-\mathbf{q}) \quad (25)$$

$$[\bar{H}, \bar{\chi}] = 0 \quad (26)$$

$$\bar{\chi} \simeq 0 \quad (27)$$

where the last equation need some explanation. Since \mathbf{R}_v and $\bar{\chi}$ do not appear in \bar{H} , $\bar{\chi}$ does not have any dynamics, just like the longitudinal part of the vector potential in a gauge theory where the hamiltonian is gauge invariant. We shall demand that $\bar{\chi} \simeq 0$ which means $\bar{\chi}$ will vanish within correlation functions. (Since $\bar{\chi}$ commutes with \bar{H} , this is a first class constraint preserved by the equations of motion.)

Our equations above are good for all Jain fractions. Similar equation for the case where the vortex and CF had equal and opposite charges were written down independently by Pasquier and Haldane¹⁴ and extended and exploited by Read¹⁵ to address the $\nu = \frac{1}{2}$ problem.

III. PUTTING THE HAMILTONIAN THEORY TO WORK

The rationale for working with the CF was to get a unique ground state for the HF approximation.

There are at least two good reasons to expect that the naive HF result will require fairly strong corrections. First, if we compute the matrix element of the projected electron density between any two HF states, the answer will be linear in q , whereas in the exact theory, we know that within the LLL it must go as q^2 as per Kohn's theorem¹⁶. Secondly, as $ql \rightarrow 0$, the projected electronic density has unit contribution from each CF while we would like it to be $e^*/e = \frac{1}{2p+1} = 1 - c^2$. Evidently the HF result will receive strong corrections that will renormalize these quantities till they are in line with these expectations. These renormalization will occur once we pay attention to the constraint $\bar{\chi} \simeq 0$.

Now Baym and Kadanoff¹⁷ have a procedure for improving the HF state with additional diagrammatic corrections (ladder sums) to enforce conservation laws. (The non-conservation comes from using Hartree-Fock self-energies for propagators while using bare vertices in the one-loop response functions, in violation of Ward-identities.)

For $\nu = \frac{1}{2}$ Read¹⁵ showed that this procedure restores Kohn's theorem, exhibits the overdamped mode, reveals a dipolar structure for density-density correlations and yield a compressible state. Murthy¹⁸ has used it to calculate density-density correlations in gapped fractions. We are currently using it to perform a comparative study of the fractions $\nu = \frac{1}{2}$ and $\frac{5}{2}$. In general this route must be followed whenever the constraint (or gauge invariance under the transformations generated by $\bar{\chi}$) is important.

A. Preferred charge: a short cut to the constraint

We found that in many problems where there is a large enough gap, temperature or disorder there is a short-cut to implementing the constraint in the infrared limit. We discuss this in some detail, for unlike the Baym-Kadanoff route, this one is peculiar the FQHE and we do not fully understand why it works or how we could give a better interpretation for it.

Suppose, in the hamiltonian and elsewhere, we replace $\bar{\rho}(\mathbf{q})$ by the *preferred combination*

$$\bar{\rho}^p(\mathbf{q}) = \bar{\rho}(\mathbf{q}) - c^2 \bar{\chi}. \quad (28)$$

In an exact calculation it make no difference to the computation of anything physical whether the coefficient in front of $\bar{\chi}$ is zero or $-c^2$ or anything else since $\bar{\chi}$ is essentially zero.

On the other hand in the HF approximation (which does not respect $\bar{\chi} \simeq 0$) it certainly matters what coefficient we place in front of $\bar{\chi}$. The preferred combination $\bar{\rho}^p(\mathbf{q})$ stands out as the sum of the electronic and vortex charge densities. But the reason we are forced to use is that it helps us avoid violating Kohn's Theorem within simple HF.

Consider its expansion in powers of ql :

$$\bar{\rho}^p = \sum_j e^{-i\mathbf{q}\cdot\mathbf{r}_j} \left(\frac{1}{2p+1} - il^2\mathbf{q} \times \mathbf{\Pi}_j + 0 \cdot (\mathbf{q} \times \mathbf{\Pi}_j)^2 + \dots \right). \quad (29)$$

- The transition matrix elements are now of order q^2 between HF states because coefficient of \mathbf{q} is proportional to the CF guiding center coordinate $\mathbf{r} - l^{*2}\hat{\mathbf{z}} \times \mathbf{\Pi}$ with no admixture of the CF cyclotron coordinate. This is more transparent if we use \mathbf{R} and $\boldsymbol{\eta}$ to write

$$\begin{aligned} \bar{\rho}^p(\mathbf{q}) &= (1 - i\mathbf{q} \cdot (\mathbf{R} + c\boldsymbol{\eta}) + \dots) - c^2(1 - i\mathbf{q} \cdot (\mathbf{R} + \boldsymbol{\eta}/c) + \dots) \\ &= (1 - c^2)(1 - i\mathbf{q} \cdot \mathbf{R} + \mathcal{O}(q^2)). \end{aligned} \quad (30)$$

The choice of $(-c^2)$ as the coefficient of $\bar{\chi}$ in $\bar{\rho}^p(\mathbf{q})$, uniquely determined by compliance with Kohn's Theorem, is also the one that leads to two important collateral benefits:

- The electronic charge density associated with $\bar{\rho}^p(\mathbf{q})$ is $1 - c^2 = e^*/e$.
- We see from Eqn. (29) that when $\nu = \frac{1}{2}$, the preferred density couples to an external electric field like a dipole of size $d^* = l^2\hat{\mathbf{z}} \times \mathbf{p}$ giving a precise operator expression of Read's picture¹².

The hamiltonian $\bar{H}(\bar{\rho}^p)$ is *weakly gauge invariant*, that is

$$\left[\bar{H}(\bar{\rho}^p), \bar{\chi} \right] \simeq 0 \quad (32)$$

where the $\simeq 0$ symbol means that it vanishes in the subspace obeying $\bar{\chi} = 0$. Thus neither $\bar{H}(\bar{\rho}^p)$ nor $\bar{\rho}^p$ will mix physical and unphysical states.

The significance of $\bar{H}(\bar{\rho})$ is the following. If the constraint $\bar{\chi} = 0$ is imposed *exactly*, there are many equivalent hamiltonians depending on how $\bar{\chi}$ is insinuated into it. However, in the HF *approximation*, these are not equivalent and $\bar{H}(\bar{\rho}^p)$ best approximates, between HF states and at long wavelengths, the true hamiltonian between true eigenstates. In contrast to a variational calculation where one searches among trial states for an optimal one, here the HF states are the same for a class of hamiltonians (where $\bar{\chi}$ is introduced into \bar{H} in any rotationally invariant form), and we seek the best hamiltonian, which happens to be $\bar{H}(\bar{\rho}^p)$ since encodes the fact that every electron is accompanied by a correlation hole of some sort, which leads to the correct e^* , d^* , and obeys the all important Kohn's theorem (q^2 matrix elements for the density projected to the LLL.)

The preferred charge $\bar{\rho}^p(\mathbf{q})$ and preferred hamiltonian $\bar{H}(\bar{\rho}^p)$ have been used to compute gaps, finite temperature response functions (polarization, NMR rates) and even the effect of disorder. The results are in reasonable agreement (10 – 20%) with computer simulations and real data³.

Note that when we use the preferred charge and hamiltonian we make no further reference to constraints, and simply carry out the Hartree-Fock approximation. This is based on the expectation that even if we found some way to include the effect of constraints, it will make no difference in the small ql region because the leading renormalization of e to e^* and suppression of q matrix elements down to q^2 that are achieved by the conserving approximation are built in here. Of course errors at larger q will corrupt the actual numbers, say for gaps.

The shortcut however fails in one important regard. For the gapless $\nu = \frac{1}{2}$ state at $T = 0$, since $\bar{\rho}^p(\mathbf{q})$ starts out linearly in \mathbf{q} , the CF couples like a dipole to the external potential, leading to a compressibility that vanishes as $q \rightarrow 0$. The only way to restore compressibility is to have some very low energy collective excitations that overcome the factors of q in the matrix elements. This was first pointed out to us by Halperin and Stern in¹⁹ who used a toy model to make their point that respecting gauge invariance (or the constraint) is crucial. They went on to give a detailed analysis of the realistic model with additional coworkers²⁰. Subsequently Read¹⁵ did the ladder sum on top of HF and obtained the overdamped mode, finite compressibility and dipolar coupling.

The reader will recall that any *simple* picture of quasiparticles, whether it be in Landau's Fermi liquid theory, or in BCS theory, is best captured by approximate and not exact descriptions. The quasiparticles are all caricatures of some exact reality and therein lies their utility. Similarly the CF in our extended formalism appears only in the HF approximation to $\bar{H}(\bar{\rho}^p)$. Recall that we brought in the coordinate \mathbf{R}_v to become the electron's partner in forming the CF. However \mathbf{R}_v was cyclic in the exact hamiltonian \bar{H} . Thus the exact dynamics never demanded that \mathbf{R}_v be bound to \mathbf{R}_e or even be anywhere near \mathbf{R}_e . However, in the HF approximation, since we wanted the right charge and transition matrix elements of the density operator (Kohn's theorem) to be manifest, we needed to replace $\bar{\rho}$ by $\bar{\rho}^p$, and trade $\bar{H}(\bar{\rho})$ for $\bar{H}(\bar{\rho}^p)$, the preferred hamiltonian. In $\bar{H}(\bar{\rho}^p)$, \mathbf{R}_v is coupled to \mathbf{R}_e . The HF approximation and this coupling go hand in hand. The exact eigenfunctions of the original \bar{H} are factorized in the analytic coordinates z_e and z_v and presumably reproduce the electronic correlations of the FQHE states. On the other hand, in the HF approximation to $\bar{H}(\bar{\rho}^p)$, the wavefunctions (e.g., p -filled LL's) mix up z_e and z_v , and $\bar{H}(\bar{\rho}^p)$, the preferred hamiltonian, dynamically couples \mathbf{R}_e and \mathbf{R}_v . The net result is that, at least at long wavelengths, these two wrongs make it right and mimic what happens in the exact solution.

Another advantage of $\bar{H}(\bar{\rho}^p)$ is that it gives an approximate formula for m^* originating entirely from in-

interactions. This is best seen at $\nu = \frac{1}{2}$. When we square $\bar{\rho}^p$ (Eqn. (29)), we get a double sum over particles whose diagonal part is the one particle (free-field) term:

$$H_{\nu=\frac{1}{2}}^0 = 2 \sum_j \int \frac{d^2q}{4\pi^2} \sin^2 \left[\frac{\mathbf{q} \times \mathbf{k}_j l^2}{2} \right] v(q) e^{-q^2 l^2/2}. \quad (33)$$

This is not a hamiltonian of the form $k^2/2m^*$. However if the potential is peaked at very small q , we can expand the sine and read off an approximate $1/m^*$

$$\frac{1}{m^*} = \int \frac{q dq d\theta}{4\pi^2} [(\sin^2 \theta) (ql)^2] v(q) e^{-q^2 l^2/2} \quad (34)$$

which has its origin in electron-electron interactions. However we can do more: we have the full H_0 as well as the interactions. The point to emphasize is that H is not of the traditional form $(p^2/2m + V)$ and that there is no reason it had to be. This proves crucial in understanding the data from Ref. V.

IV. CONCLUSIONS AND SUMMARY

The trial wavefunctions tell us that the quasiparticle of the FQHE is an electron bound to vortices, that this entity which resides in the LLL of electrons has charge e^* , mass and dynamics generated entirely by the interactions and no memory of m . Here we show one way to implement that concept within a Hamiltonian and commuting constraints. It consists of complementing \mathbf{R}_e , the guiding center coordinate of the electron with \mathbf{R}_v , the guiding

center coordinate of an entity that has the same charge as the vortex. The two guiding centers fit nicely into the Hilbert space of the composite fermion which sees a field just right to fill p LL's or the Fermi sea, paving the way for a HF calculation. But one finds that naive HF violates the constraint, violates Kohn's theorem and describes a particle with the electronic charge e and not e^* .

We describe two ways to fix it. One is the standard ploy of using a conserving approximation in which particle-hole ladder graphs restore the Ward identity. This is quite involved but necessary whenever physics at very small ω and q is to be faithfully described and gauge invariance is crucial.

The second method, which seems to have no analog outside of the FQHE is to add to the electronic density in the Hamiltonian a judicious amount of the constraint to salvage Kohn's theorem within the naive HF calculation. This ends up producing the right quasiparticle charge and dipole moment. It describes the data on gaps, relaxation rate and polarization rather well at finite frequency, wave vector, temperature and even disorder. At this moment we do not have a deeper understanding of why it works as well as it does or if there is a better way to introduce it into the formalism. I offer it as a challenge to the readers, especially Dieter, to stimulate his aging brain.

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